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<https://www.obspm.fr/the-poincare-dodecahedral-space-model-gains.html>

# The Poincaré Dodecahedral Space model gains support to explain the shape of space



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**An international team of cosmologists, led by a researcher from Paris Observatory, has improved the theoretical pertinence of the Poincaré Dodecahedral Space (PDS) topology to explain some observations of the Cosmic Microwave Background (CMB). In parallel, another international team has analyzed with new techniques the last data obtained by the WMAP satellite and found a topological signal characteristic of the PDS geometry.**

The last fifteen years have shown considerable growth in attempt to determine the global shape of the universe, i.e. not only the curvature of space but also its topology. The « concordance » cosmological model which now prevails describes the universe as a « flat » (zero-curvature) infinite space in eternal, accelerated expansion. However, the data delivered between 2003 and 2006 by the NASA satellite WMAP, which produced a full-sky, high resolution map of the Cosmic Microwave Background Radiation (CMB), yield a very poor fit to the concordance model at large angular scales. They rather tend to favor a finite, positively curved space, and provide hints about a multiply-connected topology.

The CMB is the relics of the radiation emitted soon after the Big Bang. It is observed on the so-called last scattering surface (LSS), a sphere of radius about 50 billion light-years around us. The tiny temperature fluctuations observed on the LSS may be decomposed into a sum of spherical harmonics, much like the sound produced by a music instrument may be decomposed into ordinary harmonics. The relative amplitudes of each spherical harmonics determine the power spectrum, which is a signature of the geometry of space and of the physical conditions which prevailed at the time of CMB emission.

Now, cosmic topology predicts that a space which is smaller than the LSS cannot contain vibrational modes larger than the space itself. This should lead to a cutoff of power in statistics representing these fluctuations, above which power should drop to zero. The predicted cutoff in large scale power has precisely been observed by the 2003-2006 WMAP all-sky survey.

Motivated by indications that the Universe may have positive curvature, and calculating large-angle vibrational harmonics to simulate the power spectrum, some authors of the present study [ref. 2] had already argued in October 2003 that the multiply-connected Poincaré dodecahedral space (PDS) topology was favoured by the WMAP data relative to an infinite, simply connected flat space.



**Figure 1 : A gauche : L'espace dodécaédrique de Poincaré peut se décrire comme l'intérieur d'un dodécaèdre sphérique tel que, si l'on « sort » par une face pentagonale, on « rentre » immédiatement par la face opposée après une rotation de  $36^\circ$ . Un tel espace est donc fini, bien que sans frontière ni bord, de sorte**

que l'on peut y voyager indéfiniment sans obstacle. Au centre : Vue depuis l'intérieur de PDS perpendiculairement à une face pentagonale. L'observateur a l'illusion de vivre dans un espace 120 fois plus grand, construit comme une mosaïque de dodécaèdres empilés dont les images se répètent comme dans une galerie des glaces. A droite : Vue depuis l'intérieur de PDS dans une direction arbitraire, calculée par le programme CurvedSpaces et montrant des images multiples de la Terre obtenues par mirage topologique (d'après Jeff Weeks). Cliquer sur l'image pour l'agrandir

The PDS model has since been studied in more mathematical details by several teams all around the world. In the most recent study, Luminet and co-workers [ref. 1] calculated 1,7 billion vibrational modes of PDS to simulate more accurately the power spectrum, from large to small angular scales. They found that the maximal repression of the quadrupole signal, as found in the data, requires an optimal total density of  $\hat{\rho}_{\text{tot}} = 1.018$  (see note 1). Their predicted PDS power spectrum then remarkably agrees with the observed one (figure 2).

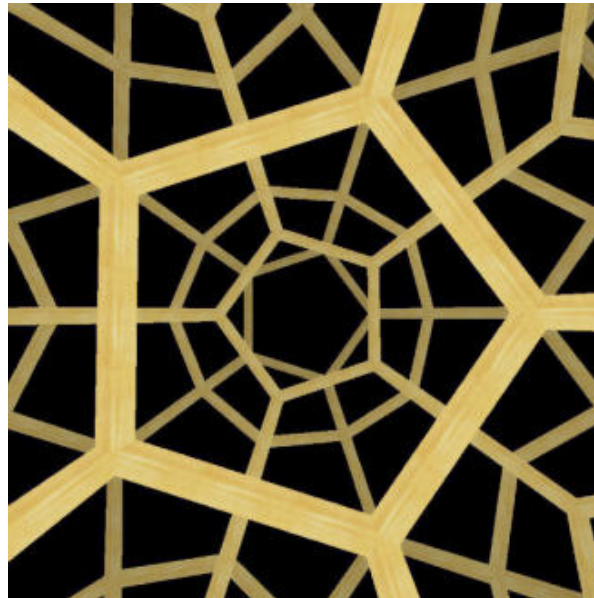


Figure 2 : Spectres de puissance comparés pour les données expérimentales de WMAP (barres d'erreur verticales), pour le modèle théorique de concordance tel que  $\hat{\rho}_{\text{tot}} = 1.000$ ,  $\hat{\rho}_{\text{mat}} = 0.27$  et  $h = 0.70$  (courbe en pointillés) et pour le modèle PDS tel que  $\hat{\rho}_{\text{tot}} = 1.018$  (courbe pleine). Cliquer sur l'image pour l'agrandir

## Circle signature

If physical space is smaller than the observed space inside the LSS sphere, there must be particular correlations in the CMB, namely pairs of « matched » circles along which temperature fluctuations should be the same, as they would represent the same physical points but observed from different directions due to topological lensing. As a definite signature of the underlying topology, the PDS model predicts six pairs of antipodal matched circles with a relative phase of  $36^\circ$ . To test this prediction, the team [ref.1] has simulated CMB temperature fluctuations maps in the PDS topology and checked the presence of the expected circles-in-the-sky (figure 4).



**Figure 3 : Simulation de la surface de dernière diffusion et de ses plus proches copies dans la topologie PDS. Comme le volume de l'espace PDS ne représente que 80% du volume de la sphère LSS, celle-ci s'auto-intersecte selon six paires antipodales de cercles homologues, qui représentent les mêmes points de l'espace à une rotation de  $36^\circ$  près. Cliquer sur l'image pour l'agrandir**

Now the crucial question is : are these pairs of matched circles present in the real WMAP data ? Three different teams (from USA, Germany and Poland) have addressed the problem in the recent years, using various statistical indicators and massive computer calculations. No clear answer presently emerges, because the expected positive correlation signal from matched pairs is spoiled by various cosmological effects, astrophysical foregrounds and instrumental effects that constitute noise.

Thus, another international team of cosmologists [ref. 3] lead by B. Roukema of Nicolaus Copernicus University in Torun, Poland (formerly at Paris-Meudon Observatory), has reanalyzed the WMAP data with new statistical tools. They have shown that cross-correlations of temperature fluctuations on multiple copies of the LSS imply a highly cross-correlated PDS symmetry with a phase of approximately  $36^\circ$  for the matched circles. By determining the position of such circles, they were even able to fix the space orientation of the fundamental dodecahedron relative to the CMB frame. The chance of this occurring in the simply-connected flat model is only 7%.

## Conclusion

Do we really live in a Poincaré Dodecahedral Space ? Further constraints either for or against the model are certainly still needed, but the evidence in favour of a PDS-like signal in the WMAP data does seem to be cumulating. To clarify the issue, new data from the future European satellite Planck Surveyor (launch scheduled in July 2008) are eagerly

expected.

Note 1 The mass-energy density parameter  $\hat{\Omega}_{\text{tot}}$  characterizes the contents (matter and all forms of energy) of the universe. The curvature of space depends on the value of this parameter. If  $\hat{\Omega}_{\text{tot}}$  is greater than 1, then space curvature is positive and geometry is spherical ; if  $\hat{\Omega}_{\text{tot}}$  is smaller than 1 the curvature is negative and the geometry is hyperbolic ; eventually  $\hat{\Omega}_{\text{tot}}$  is strictly equal to 1 and space is Euclidian.

## References

- [1] A new analysis of the Poincaré dodecahedral space model, S. Caillerie, M. Lachièze-Rey, J.-P. Luminet, R. Lehoucq, A. Riazuelo & J. Weeks, *Astron. Astrophys.* 2007, 476, 691.696.
- [2] Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background, J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq & J.-P. Uzan, *Nature*, 2003, 425, 593-595.
- [3] The optimal phase of the generalised Poincaré Dodecahedral Space hypothesis implied by the spatial cross-correlation function of the WMAP sky maps, B. Roukema, Z. Bulinski, A. Szaniewska & N. Gaudin, 2008 [arXiv:0801.0006v1] and J.- P. Luminet : *The wraparound universe*, AK Peters, New York, 2008 (English translation of *L'Univers chiffonné*, 2nd edition, Gallimard, Paris, 2005).

## Contact

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